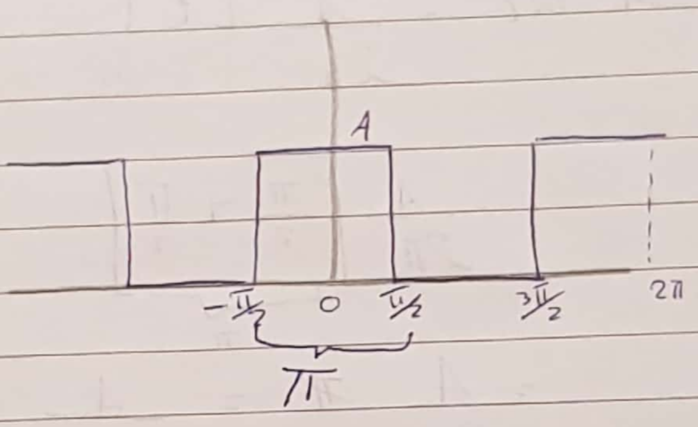
Forsrier Series:-

Compute the First componont of the trignometric Fovrier senies Assume $\omega = 1$



Even signal → $b_n = 0$

نجد a_n و a_0

$\omega = 1 \quad T = 2\pi$

$$x(t) = \begin{cases} A, & -\frac{\pi}{2} \le t \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le t \le \frac{3\pi}{2} \end{cases}$$

$$a_n = \frac{A}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{-n\pi}{2}\right)\right]$$

$$a_n = \frac{A}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right)\right]$$

$$a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

when $n=1$

$$a_1 = \frac{2A}{\pi} \sin\left(\frac{\pi}{2}\right) = \frac{2A}{\pi}$$

$$a_2 = \frac{2A}{2\pi} \sin(\pi) = 0$$

$$a_3 = \frac{2A}{3\pi} \sin\left(\frac{3\pi}{2}\right) = \frac{-2A}{3\pi}$$

$$a_4 = \frac{2A}{4\pi} = \sin(2\pi) = 0$$

قانون ①

$$a_0 = \frac{\text{Area over one } T}{T} = \frac{A\pi}{2\pi} = \frac{A}{2}$$

قانون ②

$$a_0 = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} A\,dt = \frac{A}{2\pi}\left[t\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{A}{2\pi}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] \quad (\pi)$$

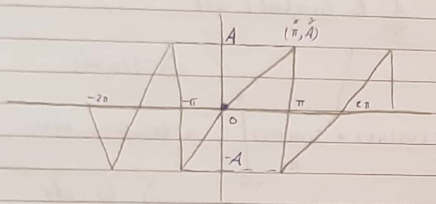
$$= \frac{A}{2\pi}\pi = \frac{A}{2} \; /\!\!/$$

$$\left\{a_n = \frac{2}{T}\int_0^T x(t)\cdot\cos(n\omega t)\cdot dt\right\}$$ قانون a_n

$$a_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} (A)\cdot\cos(nt)\cdot dt$$

$$= \frac{A}{\pi}\left[\frac{\sin(nt)}{n}\right]_{-\pi/2}^{\pi/2}$$

نفس السؤال الي قبله



The signal is odd $a_0 = 0$, $a_n = 0$

$\omega = 1 \rightarrow T = 2\pi$

$x(t) = \frac{A}{\pi} t \qquad -\pi \le t \le \pi$

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - 0}{x - 0} = \frac{A - 0}{\pi - 0}$

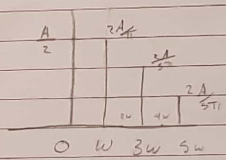
$\frac{y}{x} = \frac{A}{\pi} \rightarrow y = \frac{A}{\pi} x \rightarrow x(t) = \frac{A}{\pi} t$

$a_5 = \frac{2A}{5\pi} \sin\left(\frac{5\pi}{2}\right) = \frac{2A}{5\pi}$

$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t)\right)$

من قانون   من قانون   من قانون

$x(t) = \frac{A}{2} + \underbrace{\frac{2A}{\pi}\cos(t)}_{a_1} - \underbrace{\frac{2A}{3\pi}\cos(3t)}_{a_3} + \underbrace{\frac{2A}{5\pi}\cos(5t)}_{a_5}$



$b_1 = \frac{-2A}{\pi} \cos(\pi) = \frac{2A}{\pi}$

$b_2 = \frac{-2A}{2\pi} \cos(2\pi) = \left| -\frac{A}{\pi} \right|$

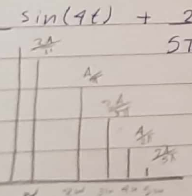
$b_3 = \frac{-2A}{3\pi} \cos(3\pi) = +\frac{2A}{3\pi}$

$b_4 = \frac{-2A}{4\pi} \cos(4\pi) = \left| -\frac{A}{2\pi} \right|$

$b_5 = \frac{-2A}{5\pi} \cos(5\pi) = \frac{2A}{5\pi}$

$x(t) = \frac{2A}{\pi} \sin(t) - \frac{A}{\pi} \sin(2t) + \frac{2A}{3\pi} \sin(3t)$

$- \frac{A}{2\pi} \sin(4t) + \frac{2A}{5\pi} \sin(5t)$



ثم رسم
لازم قيمة مطلقة
$|X(t)|^2$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \left[\frac{A}{\pi}\right] t \sin(nt) \, dt$

~~$\int u \, dv$~~

$b_n = \frac{A}{n\pi^2} \left[-t\cos(nt) + \frac{\sin(nt)}{n} \right]_{-\pi}^{\pi}$

$\int u \, dv$
$u \cdot v - \int v \, dv$

$b_n = \frac{A}{n\pi^2} \left[-\pi \cos(n\pi) + \frac{\sin(n\pi)}{n} \right]$

$-\pi \cos(-n\pi) - \frac{\sin(-n\pi)}{n}$

$u = t \quad dv = \sin(nt)\, dt$

$du = dt \quad v = -\frac{\cos(nt)}{n}$

$-\frac{t\cos(nt)}{n} + \int \frac{\cos(nt)}{n} dt$

$b_n = \frac{A}{n\pi^2} \left[-\pi \cos(n\pi) - \pi \cos(n\pi) \right]$

$-2\pi \cos(n\pi)$

$b_n = \frac{-2A\pi}{n\pi^2} \cos(n\pi)$

$b_n = \frac{-2A}{n\pi} \cos(n\pi)$

$$a_n = \frac{2}{T} \int_{-T}^{T} F(t) \cos(n\omega t)\,dt$$

$$a_n = \frac{1}{\pi} \left[\int_0^{\pi/2} 3\cos(nt)\,dt + \int_{\pi/2}^{2\pi} \cos(nt)\,dt \right]$$

$$a_n = \frac{1}{n\pi} \left[3\sin\left(\frac{n\pi}{2}\right) - 3\sin(n(0)) + \sin(2\pi n) - \sin\left(\frac{\pi}{2} \cdot n\right) \right]$$

$$a_n = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

~~$b_n = \frac{1}{\pi} \left[\int_0^{\pi/2} 3\cos(nt)\,dt + \int_{\pi/2}^{2\pi} \cos(nt)\,dt \right]$~~

$$b_n = \frac{1}{\pi} \left[\int_0^{\pi/2} 3\sin(nt)\,dt + \int_{\pi/2}^{2\pi} \sin(nt)\,dt \right]$$

$$b_n = \frac{-1}{n\pi} \left[3\cos(nt) \Big|_0^{\pi/2} + \cos(nt) \Big|_{\pi/2}^{2\pi} \right]$$

$$b_n = \frac{-1}{n\pi} \left[3\cos\left(\frac{n\pi}{2}\right) - \underbrace{3\cos(0)}_{(-3)} + \cos(2\pi n) - \underbrace{\cos\left(n\frac{\pi}{2}\right)}_{(-1)} \right]$$

$$b_n = \frac{2}{n\pi} \left[1 - \cos\left(\frac{nt}{2}\right) \right]$$

$$F(t \pm 2\pi) = \begin{cases} 3, & 0 \le t \le \frac{\pi}{2} \\ 1, & \frac{\pi}{2} \le t \le 2\pi \end{cases}$$

$$a_0 = \frac{3 * \frac{\pi}{2} + 1 * \frac{3\pi}{2}}{2\pi} = \frac{3}{2}$$

or

$$\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} A\,dt = \frac{A}{2\pi} \int_{-\pi/2}^{\pi/2} dt = \frac{A}{2\pi} \left[t \right]_{-\pi/2}^{\pi/2} = \frac{A}{2\pi} \left[\underbrace{\frac{\pi}{2} + \frac{\pi}{2}}_{\pi} \right]$$

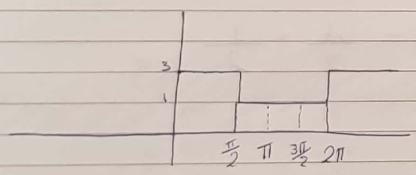
$$= \frac{A}{2\pi} * \pi = \frac{A}{2} = \frac{3}{2}$$

$$F(t) = a_0 + \sum_{n=1}^{\infty} \cos(n\omega t - \theta_n)$$

$C_n = \sqrt{a_n^2 + b_n^2}$ $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

Example:- Find the First 5 terms of the alterte Form of the trigohmeic Forier series whene $(\omega = 1)$

$\omega = \frac{2\pi}{T}$



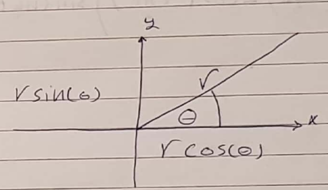
$T = 2\pi$

The signal is neither even nor odd

الاشارة لا زوجية وفردية

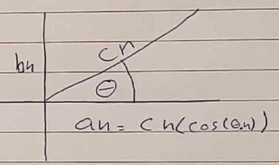
Alternative or Polar Form of trigonometric Form series:-

$$F(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$



$b_n = c_n \sin\theta_n$

نعوض في القانون



$$F(t) = a_0 + \sum_{n=1}^{\infty} \left[\underbrace{c_n(\cos\theta_n)}_{a} \cos(n\omega t) \mp \underbrace{c_n \sin(\theta_n)}_{b} \sin(n\omega t)\right]$$

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$x(t) = \frac{-2A}{5} + \sum_{n=1}^{\infty} \left[\frac{2A}{n\pi} \left(2\sin\left(\frac{2n\pi}{5}\right) - \sin\left(\frac{6n\pi}{5}\right) \right) \cos(n\omega t) \right.$$

$$\left. + \frac{2A}{n\pi} \left(\cos\left(\frac{6n\pi}{5}\right) - \cos\left(\frac{2n\pi}{5}\right) \right) \sin(n\omega t) \right]$$

$$a_n = \frac{2A}{n} \left[2\sin\left(\frac{2n\pi}{5}\right) - \sin\left(\frac{6n\pi}{5}\right) \right]$$

$$b_n = \frac{4A}{5\pi} \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(n\omega t).dt - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\sin(n\omega t).dt \right]$$

$$b_n = \frac{4A}{5\pi} \left[\left[\frac{-\cos(n\omega t)}{n\omega} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \left[\frac{2\cos(n\omega t)}{n\omega} \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right]$$

$$b_n = \frac{A}{n\pi} \left[-\cos\left(n * \frac{4}{5} * \frac{\pi}{2}\right) + \cos\left(-n * \frac{4}{5} * \frac{\pi}{2}\right) \right.$$

$$\left. + 2\cos\left(n * \frac{4}{5} * \frac{3\pi}{2}\right) - 2\cos\left(n * \frac{4}{5} * \frac{3\pi}{2}\right) \right]$$

$$b_n = \frac{A}{n\pi} \left[-\cos\left(\frac{2n\pi}{5}\right) + \cos\left(\frac{2n\pi}{5}\right) + 2\cos\left(\frac{6n\pi}{5}\right) - 2\cos\left(\frac{2n\pi}{5}\right) \right]$$

$$b_n = \frac{2A}{n\pi} \left[\cos\left(\frac{6n\pi}{5}\right) - \cos\left(\frac{2n\pi}{5}\right) \right]$$

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$$a_0 = \frac{A\pi - 2A\pi}{\frac{5\pi}{2}} = \frac{-A\pi \times 2}{5\pi} = \frac{-2A}{5}$$

$$a_n = \frac{2 \times 2}{5\pi}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\cos(n\omega t).dt - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2A\cos(n\omega t).dt\right)$$

$$a_n = \frac{4A}{5\pi}\left(\left[\frac{\sin(n\omega t)}{n\omega}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \left[\frac{2\sin(n\omega t)}{n\omega}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\right)$$

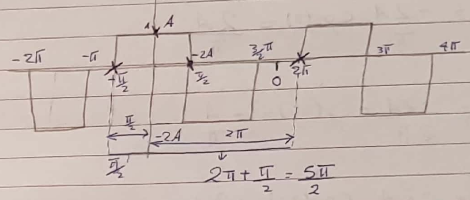
ساوه n عامل مشترك

$$a_n = \frac{4A}{n \cdot 5 \times \frac{4}{5}\pi}\left[\sin\left(n\frac{4}{5} \times \frac{\pi}{2}\right) - \sin\left(-n\frac{4}{5} \times \frac{\pi}{2}\right)\right.$$

$$\left. - 2\sin\left(n \times \frac{4}{5} \times \frac{3\pi}{2}\right) + 2\sin\left(n \times \frac{4}{5} \times \frac{\pi}{2}\right)\right]$$

$$a_n = \frac{A}{n\pi}\left[\sin\left(\frac{2n\pi}{5}\right) + \sin\left(\frac{2n\pi}{5}\right) - 2\sin\left(\frac{6n\pi}{5}\right)\right.$$

$$\left. + 2\sin\left(\frac{2n\pi}{5}\right)\right]$$



The signal is neither even or odd

$$T = \frac{5\pi}{2} \qquad \omega = \frac{2\pi \times 2}{5\pi} = \frac{4}{5}$$

$$x(t) = \begin{cases} A, & \frac{-\pi}{2} \le t \le \frac{\pi}{2} \\ -2A, & \frac{\pi}{2} \le t \le \frac{3}{2}\pi \\ 0, & \frac{3\pi}{2} \le t \; 2\pi \end{cases}$$

(b)

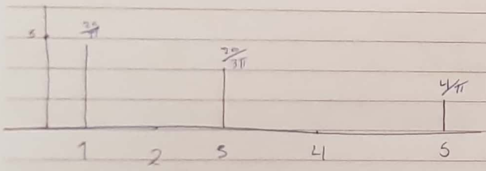
$b_1 = \frac{20}{\pi} \sin\left(\frac{\pi}{2}\right) = \frac{20}{\pi} \rightarrow 90^\circ$

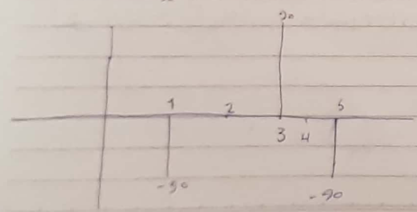
$b_2 = \frac{10}{\pi} \sin(\pi) = 0$

$b_3 = \frac{20}{3\pi} \sin\left(\frac{3\pi}{2}\right) = \frac{-20}{3\pi} \rightarrow -90^\circ$

$b_4 = \frac{5}{\pi} \sin(2\pi) = 0$

$b_5 = \frac{4}{\pi} \sin\left(\frac{5\pi}{2}\right) = \frac{4}{\pi} \rightarrow 90^\circ$





* A Fourier series of a signal is given as:-

$$x(t) = 5 + \sum_{n=1}^{\infty} \frac{20}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin(4nt)$$

(a) Find the Fundamental Period and Average Value

(b) sketch the spectrum For the First 5 terms

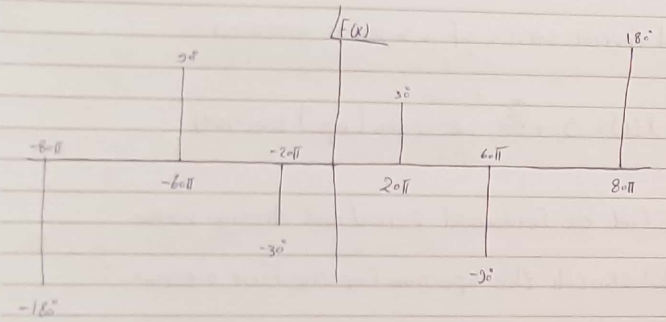
(c) Is the signal even or odd or naither

(a) $\sin(4nt) \rightarrow \sin(n\omega t)$

$\therefore \omega = 4 \rightarrow T = \frac{2\pi}{4} = \frac{\pi}{2}$ sec

$a_0 = 5$

(c) becaus both terms a_0 and b_n are present The signal is naither even nor odd



خطوة ليست مهمة

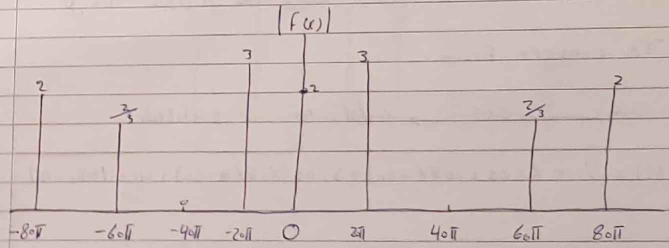
$\omega = 2\pi$

$T_1 = \frac{2\pi}{20\pi} = \frac{1}{10}$ | $T_2 = \frac{2\pi}{60\pi} = \frac{1}{30}$ | $T_3 = \frac{2\pi}{80\pi} = \frac{1}{40}$

$\frac{T_1}{T_2} , \frac{T_2}{T_3} , \frac{T_3}{T_1}$

$L.C.M = \frac{L.C.M}{G.C.F} = \frac{1.1.1}{10.30.40} = \frac{1}{10}$

خطوة مهمة [illegible]



* Draw the two sided line spectrum of :-

$F(t) = 2 + 6\cos(20\pi t + 30) + 3\sin(60\pi t) - 4\cos(80\pi t)$

* C_n must be positive on the form of $\cos(n\omega t - \theta_n)$

① To convert from

$-C_n \rightarrow +C_n$ → add 180° or subtract

كل $-C_n$ تحول الى $+C$ وننقص او نزيد 180°

② To convert from

sin → cos → add 90° or subtract

* $F(t) = 2 + 6\cos(20\pi t + 30) + 3\cos(60\pi t - 90) + 4\cos(80\pi t + 180)$

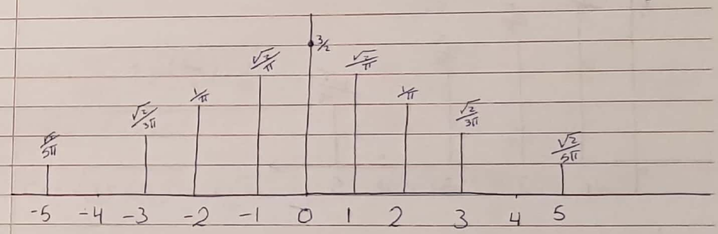
Double side spectrum DSS

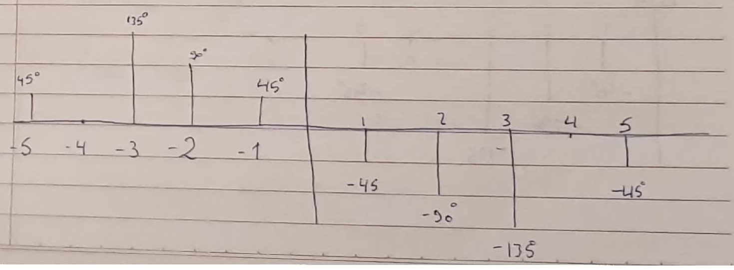
① $|F(x)|$ is even function

② $\angle F(x)$ is odd function

③ C_n for DSS is equal to $\frac{1}{2} C_n$ [SSS]

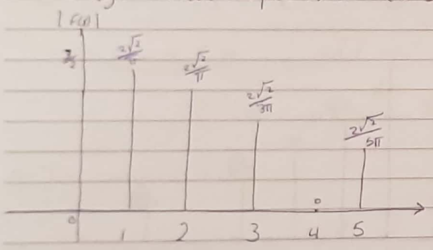
في DSS تاخذ قيم SSS ونقسمه على 2

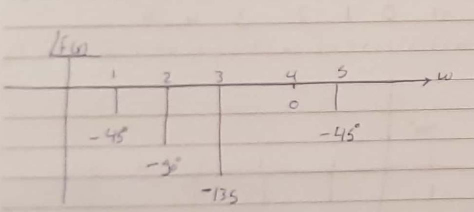




$$F(t) = \frac{3}{2} + \frac{2\sqrt{2}}{\pi}\cos(t-45) + \frac{2}{\pi}\cos(2t-90) + \frac{2\sqrt{2}}{3\pi}\cos(3t-135) + \frac{2\sqrt{2}}{5\pi}\cos(5t-45)$$

Single side spectrum (SSS)





$a_1 = \frac{2}{\pi}\sin\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$ (1) $b_1 = \frac{2}{\pi}\left[1-\cos\left(\frac{\pi}{2}\right)\right] = \frac{2}{\pi}$

$a_2 = \frac{2}{2\pi}\sin(\pi) = 0$ (0) $b_2 = \frac{1}{\pi}\left[1-\cos(\pi)\right] = \frac{2}{\pi}$

$a_3 = \frac{2}{3\pi}\sin\left(\frac{3\pi}{2}\right) = \frac{-2}{3\pi}$ (-1) $b_3 = \frac{2}{3\pi}\left[1-\cos\left(\frac{3\pi}{2}\right)\right] = \frac{2}{3\pi}$

$a_4 = \frac{2}{4\pi}\sin(2\pi) = 0$ (0) $b_4 = \frac{1}{2\pi}\left[1-\cos(2\pi)\right] = 0$

$a_5 = \frac{2}{5\pi}\sin\left(\frac{5\pi}{2}\right) = \frac{2}{5\pi}$ (1) $b_5 = \frac{2}{5\pi}\left[1-\cos\left(\frac{5\pi}{2}\right)\right] = \frac{2}{5\pi}$

$C_n = \sqrt{a_n^2 + b_n^2}$ $\theta_n = \tan^{-1}\frac{b_n}{a_n}$

$C_1 = \sqrt{\left(\frac{2}{\pi}\right)^2 + \left(\frac{2}{\pi}\right)^2} = \frac{2\sqrt{2}}{\pi}$ $\theta_1 = \tan^{-1}\frac{2/\pi}{2/\pi} = 1 = 45^\circ$

$C_2 = \sqrt{(0)^2 + \left(\frac{2}{\pi}\right)^2} = \frac{2}{\pi}$ $\theta_2 = \tan^{-1}\frac{2/\pi}{0} = 90$??

$C_3 = \sqrt{\left(\frac{-2}{3\pi}\right)^2 + \left(\frac{2}{3\pi}\right)^2} = \frac{2\sqrt{2}}{3\pi}$ $\theta_3 = \tan^{-1}(-1) = 135$

$C_4 = 0$ $\theta_4 =$ قيمة غير معرفة

$C_5 = \sqrt{\left(\frac{2}{5\pi}\right)^2 + \left(\frac{2}{5\pi}\right)^2} = \frac{2\sqrt{2}}{5\pi}$ $\theta_5 = 45^\circ$

$$D_4 = \frac{A}{4\pi}\left[\frac{1+1}{-3} + \frac{1+1}{5}\right]$$

$$D_4 = \frac{A}{4\pi}\left[\frac{-2}{3} + \frac{2}{5}\right] = \frac{A}{4\pi}\left[\frac{-10+6}{15}\right] = \frac{-A}{15\pi}$$

$$|D_4| = \frac{A}{15\pi} \qquad \theta_4 = 180^\circ - 0^\circ = 180^\circ$$

$$m = a + jb$$

$$|m| = \sqrt{a^2+b^2}$$

$$\angle m = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta_4 = \tan^{-1}\frac{0}{-\frac{A}{15\pi}} = 180^\circ$$

$$D_0 = \frac{-A}{4\pi}\left[\frac{-\cos(0)-1}{1} \quad \frac{-\cos(0)-1}{1}\right]$$

$$D_0 = \frac{-A}{4\pi}\left[-2-2\right] = \frac{A}{\pi}$$

$$D_2 = \frac{A}{4\pi}\left[-2 + \frac{2}{3}\right]$$

$$D_2 = \frac{-A}{3\pi}, \quad |D_2| = \frac{A}{3\pi}$$

$$\theta_2 = 180^\circ - 0^\circ = 180^\circ$$

$$D_3 = \frac{A}{4\pi}\left[\frac{\cos(3\pi)+1}{-2} + \frac{\cos(3\pi)+1}{4}\right]$$

$$D_3 = \frac{A}{4\pi}\left[\frac{0}{-2} + \frac{0}{4}\right] = 0$$

When $n=1$

$$D_1 = \frac{A}{4\pi j} \int_0^{\pi} (1 - e^{-2jt}).dt$$

$$D_1 = \frac{A}{4\pi j} \left[\pi + \frac{e^{-2jt}}{2j} \Big|_0^{\pi} \right]$$

$$D_1 = \frac{A}{4\pi j} \left[\pi + \frac{e^{-2j\pi} - 1}{2j} \right]$$

$$D_1 = \frac{A}{4\pi j} \left[\pi + \frac{1-1}{2j} \right]$$

$$D_1 = \frac{A}{4j} \quad , \quad \frac{A}{4}$$

$$D_1 = \frac{-Aj}{4} \quad , \quad |D_1| = \sqrt{(0)^2 + \left(\frac{-A}{4}\right)^2}$$

$$|D_1| = \sqrt{\frac{A^2}{16}} = \frac{A}{4}$$

$$\theta_1 = \frac{0°}{90} = 0° - 90° = -90°$$

$$D_n = \frac{A}{4\pi j} \int_0^{\pi} \left(e^{(1-n)jt} - e^{-(1+n)jt}\right).dt$$

$$D_n = \frac{A}{4\pi j} \left[\frac{e^{(1-n)jt}}{(1-n)j} \Big|_0^{\pi} + \frac{e^{-(1+n)jt}}{(1+n)j} \Big|_0^{\pi} \right]$$

$-1 = j^2 = j \times j$

$$D_n = \frac{-A}{4\pi} \left[\frac{e^{(1-n)j\pi}}{(1-n)} - \frac{1}{(1-n)} + \frac{e^{-(1+n)j\pi}}{1+n} - \frac{1}{1+n} \right]$$

$$D_n = \frac{-A}{4\pi} \left[\frac{e^{j\pi} \cdot e^{-nj\pi}}{(1-n)} - \frac{1}{1-n} + \frac{e^{-j\pi} \cdot e^{-nj\pi}}{(1+n)} - \frac{1}{1+n} \right]$$

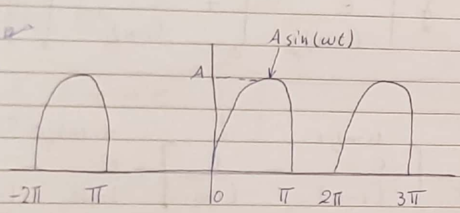
$$= \frac{-A}{4\pi} \left[\frac{-e^{-nj\pi}}{1-n} - \frac{1}{1-n} - \frac{e^{-nj\pi}}{1+n} - \frac{1}{1+n} \right]$$

$$e^{-jn\pi} = [\cos(n\pi) - j\sin(n\pi)]$$

$$D_n = \frac{-A}{4\pi} \left[\frac{-\cos(n\pi) + 1}{1-n} + \frac{\cos(n\pi) + 1}{1+n} \right] , \quad n \neq 1$$

Find the Fourier series of the following signal

Half wave rectifier :-



$T = 2\pi \qquad w = 1$

$$F(t) = \begin{cases} A\sin(t), & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi \end{cases}$$

$$D_n = \frac{A}{2\pi}\int_0^{\pi} \sin(t)\, e^{-jnt}\, dt$$

$$D_n = \frac{A}{4\pi j}\int_0^{\pi} (e^{jt} - e^{-jt})\, e^{-jnt}\, dt$$

$$F(t) = a_0 + \sum_{n=1}^{\infty} D_n e^{jnwt} + \sum_{-\infty}^{-1} D_n e^{jnwt}$$

$$\boxed{F(t) = \sum_{-\infty}^{\infty} D_n e^{jnwt}}$$

$$D_n = \frac{1}{2}\left[\frac{2}{T}\int_0^{T} f(t)\cos(nwt)\,dt - \frac{2}{T}\int_0^{T} j f(t)\sin(nwt)\,dt\right]$$

$$D_n = \frac{1}{T}\int_0^{T} f(t)\left[\cos(nwt) - j\sin(nwt)\right]dt$$

$$e^{-j\theta} = \cos(\theta) - j\sin\theta$$

$$\boxed{D_n = \frac{1}{T}\int_0^{T} f(t)\, e^{-jnwt}\, dt}$$

$$\boxed{D_0 = \frac{1}{T}\int_0^{T} f(t)\, dt}$$

Exponcutial or complex Fourier series:-

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

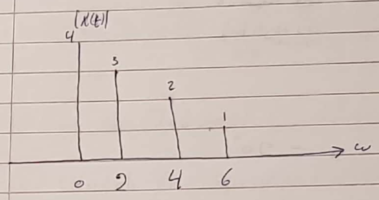
$$F(t) = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2}\left(e^{jn\omega t} + e^{-jn\omega t}\right) + \frac{b_n}{2j}\left(e^{jn\omega t} - e^{-jn\omega t}\right)$$

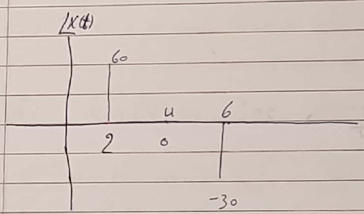
$$F(t) = a_0 + \sum_{n=1}^{\infty} \left[\left(\frac{a_n - jb_n}{2}\right) e^{jn\omega t} + \frac{a_n + jb_n}{2} e^{-jn\omega t} \right]$$

$$\frac{a_n - jb_n}{2} = D_n \quad , \quad \frac{a_n + jb_n}{2} = D_{-n} \text{ or } D^*$$

$$F(t) = a_0 + \sum_{n=1}^{\infty} D_n e^{jn\omega t} + \sum_{n=1}^{\infty} D_{-n} e^{-jn\omega t}$$

* The single side spectrum for a signal is given as:-





* حل:-

$$X(t) = 4 + 3\cos(2t+60) + 2\cos(4t) + \cos(6t-30)$$

#

$$F(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega t}$$

$$D_n = \frac{1}{T} \int_{T} F(t) e^{-jn\omega t} \cdot dt$$

$$D_n = \frac{A}{T} \int_{-\pi/k}^{\pi/k} e^{-jn\omega t} \cdot dt$$

$$D_n = \frac{A}{2\pi} \left[\frac{e^{-jnt}}{-nj} \right]_{-\pi/k}^{\pi/k} \rightarrow \frac{A}{-2\pi nj} \left[e^{-jnt} \right]_{-\pi/k}^{\pi/k}$$

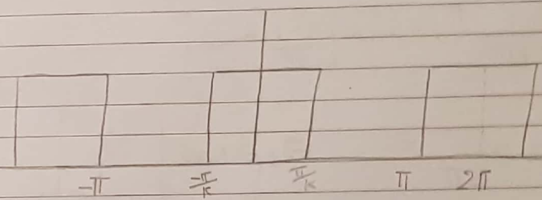
$$D_n = -\frac{A}{2\pi nj} \left[e^{-jn\frac{\pi}{k}} - e^{jn\frac{\pi}{k}} \right]$$

$$D_n = \frac{A}{n\pi} \left[\frac{e^{jn\frac{\pi}{k}} - e^{-jn\frac{\pi}{k}}}{2j} \right]$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$D_n = \frac{A}{n\pi} \sin\left(\frac{n\pi}{k}\right)$$

Example :- compute the exponential Fourier Series For the waveform where $\omega = 1$



Train of Pulses

$\omega = 1 \longrightarrow T = 2\pi$

pulse duration :- $\frac{2\pi}{k}$

$D_3 = \frac{1}{3j\pi}\left[1 - e^{-j\frac{3}{2}\pi}\right]$ → $\cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right)$

$D_3 = \frac{1}{3j\pi}\left[1 - j\right] = \left[\frac{-1}{3\pi} - \frac{j}{3\pi}\right]$

$|D_3| = \frac{\sqrt{2}}{3\pi}$ $\quad \theta_3 = \tan^{-1}\left[\frac{\frac{-1}{3\pi}}{\frac{-1}{3\pi}}\right]$

$\theta_3 = 225°$, $-135°$

$D_4 = \frac{1}{4\pi j}\left[1 - e^{-j2\pi}\right] = 0$

$D_{-1} = \frac{-1}{j\pi}\left[1 - e^{j\frac{\pi}{2}}\right]$ → $\cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right)$

$D_{-1} = \frac{j}{\pi}\left[1 - j\right]$

$D_{-1} = \frac{1+j}{\pi}$

$D_2 = \frac{1}{2\pi j}\left[1 - e^{-jn\pi}\right]$

$D_2 = \frac{1}{2\pi j}\left[1 + 1\right] = \frac{1}{\pi j}$

$|D_2| = \frac{1}{\pi}$ $\quad \theta_2 = 0° - 90 = -90°$

$$D_n = \frac{-1}{2\pi j n}\left[2e^{-jn\frac{\pi}{2}} - 2\right]$$

عامل مشترك $\boxed{-2}$

$$D_n = \frac{1}{\pi j n}\left[1 - e^{-jn\frac{\pi}{2}}\right] \quad n \neq 0$$

$$D_0 = \frac{3}{2}$$

$$D_1 = \frac{1}{j\pi}\left[1 - \boxed{e^{-j\frac{\pi}{2}}}\right] = \frac{-1}{j\pi}\left[1+j\right]$$

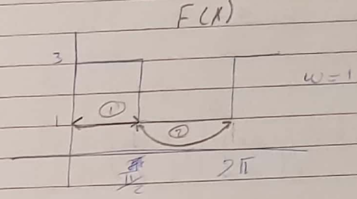
($e^{-j\frac{\pi}{2}}$ → $\cos(\frac{\pi}{2}) - j\sin(\frac{\pi}{2})$)

$$\boxed{D_1 = \frac{1-j}{\pi}} \quad , \quad \frac{-j}{\pi}\left[1+j\right] = \frac{-j}{\pi} \quad \frac{-j^2}{\pi}$$

$$|D_1| = \sqrt{\left(\frac{1}{\pi}\right)^2 + \left(\frac{-1}{\pi}\right)^2} = \frac{\sqrt{2}}{\pi}$$

$$\theta_1 = \tan^{-1}\left(\frac{\frac{-1}{\pi}}{\frac{1}{\pi}}\right) = -45^\circ$$

Ex.: ②



$T = 2\pi$

$$F(x) = \begin{cases} 3 & 0 \le t \le \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \le t \le 2\pi \end{cases}$$

$$D_n = \frac{1}{2\pi}\left[\int_0^{\frac{\pi}{2}} 3e^{-jnt}\,.dt + \int_{\frac{\pi}{2}}^{2\pi} e^{-jnt}\,.dt\right]$$

$$D_n = \frac{1}{2\pi}\left[\frac{-3e^{-jnt}}{jn}\Bigg|_0^{\frac{\pi}{2}} \quad \frac{-e^{-jnt}}{jn}\Bigg|_{\frac{\pi}{2}}^{2\pi}\right]$$

$$D_n = \frac{-1}{2\pi j n}\left[3e^{-jn\frac{\pi}{2}} - 3 + \underset{①}{e^{-jn2\pi}} - e^{-jn\frac{\pi}{2}}\right]$$

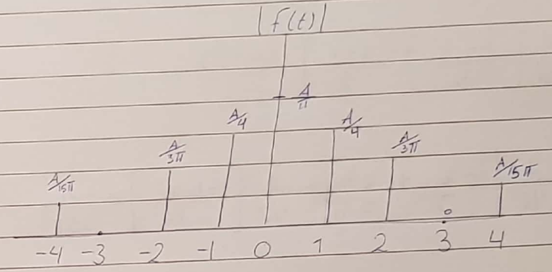
←

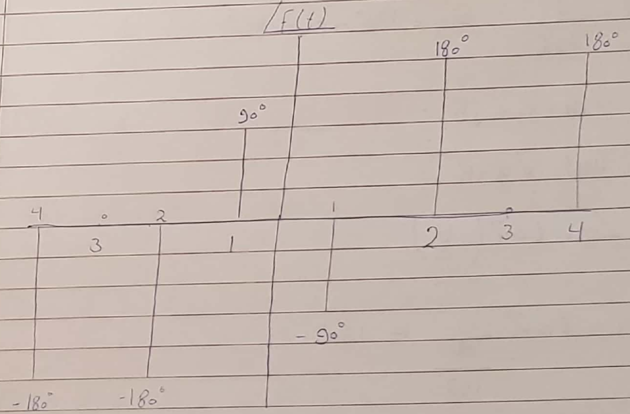
$$D_n = \frac{a_n - j b_n}{2}$$

$$|D_n| = \sqrt{\left(\frac{a_n}{2}\right)^2 + \left(\frac{-b_n}{2}\right)^2}$$

$$|D_n| = \frac{1}{2}\sqrt{(a_n)^2 + (b_n)^2}$$

$$= \frac{1}{2}|C_n|$$





~~$b_1 = \frac{1}{\pi}\left(2 - \left[\cos\left(\frac{2\pi}{3}\right) + \cos\right.\right.$~~

$$b_1 = \frac{1}{\pi}\left[2 - \left(\cos\left(\frac{2\pi}{3}\right) + \cos(\pi)\right)\right]$$

$$b_1 = 2 - \left[\frac{-1}{2} - 1\right] = 2 - \left[\frac{-3}{2}\right] = \frac{7}{2\pi}$$

$$b_2 = \frac{1}{2\pi}\left[2 - \left(\cos\left(\frac{4\pi}{3}\right) + \cos(2\pi)\right)\right]$$

$$= \frac{1}{2\pi}\left[2 - \left[-\frac{1}{2} + 1\right]\right] = \frac{3}{4\pi}$$

$$b_3 = \frac{1}{3\pi}\left[2 - \left(\cos(2\pi) + \cos(3\pi)\right)\right]$$

$$b_3 = \frac{1}{3\pi}\left[2\right] = \frac{2}{3\pi}$$

$$b_4 = \frac{1}{4\pi}\left[2 - \left(\cos\left(\frac{8\pi}{3}\right) + \cos(4\pi)\right)\right]$$

$$\left[2 - \left[-\frac{1}{2} + 1\right]\right] = \frac{3}{8\pi}$$

(IV)

$$a_n = \frac{1}{6}\left[\int_{-2}^{0} -\cos(n\omega t).dt + \int_{0}^{3}\cos(n\omega t).dt\right]$$

$$= \frac{1}{6n\omega}\left[\left[-\sin(n\omega t)\right]_{-2}^{0} + \left[\sin(n\omega t)\right]_{0}^{3}\right]$$

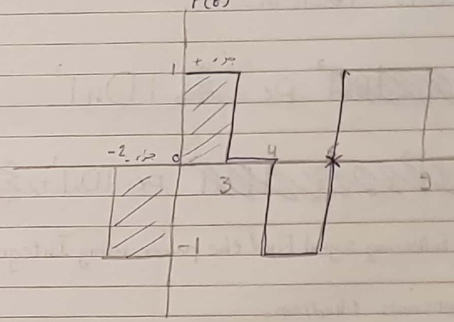
$$= \frac{1}{2\pi n}\left[-\sin(0) + \sin\left(\frac{-2n\pi}{3}\right) + \sin(n\pi) - \sin(0)\right]$$

$$= \frac{-1}{2n\pi}\left[\sin\left(\frac{2n\pi}{3}\right)\right]$$

(V) Given $b_n = \frac{1}{n\pi}\left[2 - \left(\cos\left(\frac{2n\pi}{3}\right) + \cos(n\pi)\right)\right]$

Complete the following table.

n	a_n	b_n
1	$-\frac{\sqrt{3}}{2\pi}$	~~4~~/2
2	$-\frac{\sqrt{3}}{8\pi}$	$\frac{3}{4\pi}$
3	0	
4	$\frac{-\sqrt{3}}{16\pi}$	$\frac{3}{8\pi}$



For the shown signal Find the Following

(I) The Fundamental period T

$T = 6$

(II) The Fundamental angular Freqevcy

$$\omega = \frac{2\pi}{6} = \frac{\pi}{3}$$

مساحة جزء +   مساحة جزء -

(III) $a_0 = \frac{|3*1| |-1*2|}{6} = \frac{1}{6}$

Parseval's Theorem :-

$P_t = \sum_{n=-\infty}^{\infty} |D_n|^2$

$P_t = |D_0|^2 + 2\sum_{n=1}^{\infty} |D_n|^2$

For the Following Signal Find the power using Integral method and parseval's theorem

$x(t) = 4\sin(50\pi t)$

Power of a sinusoidal signal is equal to $\left[\frac{Amplitude}{2}\right]^2$

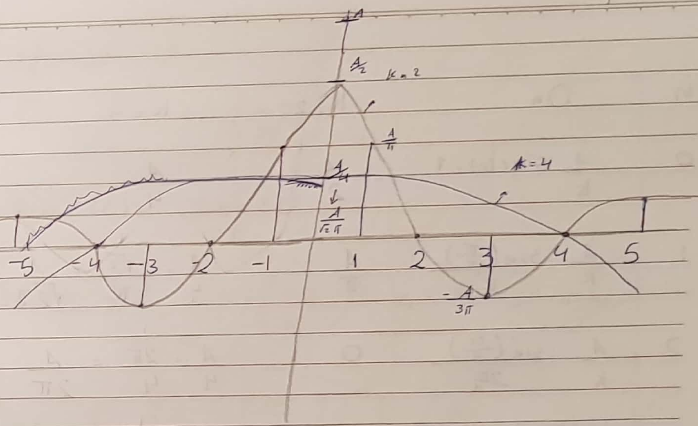
$P = \frac{(4)^2}{2} = \frac{16}{2} = 8$ watt

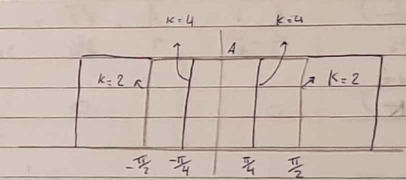
$x(t) = a_0 + \sum_{n=1}^{\infty}[a_n\cos(n\omega t) + b_n\sin(n\omega t)]$

$a_0 = 0 \rightarrow D_0 = 0$ $\quad |D_n| = \sqrt{(a_n)^2 + (b_n)^2} \cdot \frac{1}{2}$

$a_n = 0$, $b_n = 4$ $\quad \sqrt{(2)^2} * \frac{1}{2} = 2$

$D_n = \frac{a_n - jb_n}{2} = \frac{0 - j4}{2} = -2j$ | $P = 2|-2j|^2 = 2*(4)^2 = 8$ watt





when decreasing the Pulse duration Form $\pi \rightarrow \frac{\pi}{2}$ [From k=2, k=4]

the Amplitude of the spectrum will be reduced and

the width of spectrum will increase

n	D_n	k = 2	k = 4
0	$\frac{A}{k}$ sinc(0) = 1	$\frac{A}{2}$	$\frac{A}{4}$
1	$\frac{A}{k} \frac{\sin(\frac{\pi}{k})}{\frac{\pi}{k}}$	$\frac{A}{\pi}$	$\frac{A}{4} \cdot \frac{\frac{1}{\sqrt{2}}}{\frac{\pi}{4}} = \frac{A}{\sqrt{2}\pi}$
2	$\frac{A}{k} \frac{\sin(\frac{2\pi}{k})}{\frac{2\pi}{k}}$	0	$\frac{A}{4} \cdot \frac{1}{\frac{2\pi}{4}} = \frac{A}{2\pi}$
3	$\frac{A}{k} \frac{\sin(\frac{3\pi}{k})}{\frac{3\pi}{k}}$	$\frac{-A}{2} \cdot \frac{1}{\frac{3\pi}{2}}$	$\frac{A}{4} \cdot \frac{\frac{1}{\sqrt{2}}}{\frac{3\pi}{4}} = \frac{A}{3\sqrt{2}\pi}$
4	$\frac{A}{k} \frac{\sin(\frac{4\pi}{k})}{\frac{4\pi}{k}}$	0	0
5	$\frac{A}{k} \frac{\sin[\frac{5\pi}{k}]}{\frac{5\pi}{k}}$	$\frac{A}{2} \cdot \frac{1}{\frac{5\pi}{2}} = \frac{A}{\frac{5\pi}{2}}$	$\frac{A}{4} \cdot \frac{-\frac{1}{\sqrt{2}}}{\frac{5\pi}{4}}$

$\text{sinc}\left[\frac{n\pi}{k}\right] = 0 \rightarrow$ when n is a multple of k

$\frac{1\pi}{k}$ لازم تكون

$$D_n = \boxed{\frac{A}{n\pi}} \sin\left(\frac{n\pi}{k}\right)$$

$$\text{sinc}(t) = \frac{\text{sinc}(t)}{t}$$

$$D_n = \boxed{\frac{k}{k}} \cdot \boxed{\frac{A}{n\pi}} \cdot \sin\left(\frac{n\pi}{k}\right)$$ ضرب في k وقسمة على k

$$D_n = \frac{A}{k} \boxed{\frac{\sin(\frac{n\pi}{k})}{\frac{n\pi}{k}}} \quad \times \frac{k}{n\pi}$$

$$D_n = \frac{A}{k} \text{sinc}\left(\frac{n\pi}{k}\right)$$ حل السؤال الى هنا

زايد

sinc(0) = 1

$$\lim_{t \to \infty} \frac{\sin(t)}{t} = \frac{0}{0} \qquad \lim_{t \to 0} \frac{\cos(t)}{1}$$

undefined quit

$$= \frac{1}{1} = (1)$$